Noise and the frontier of quantum supremacy

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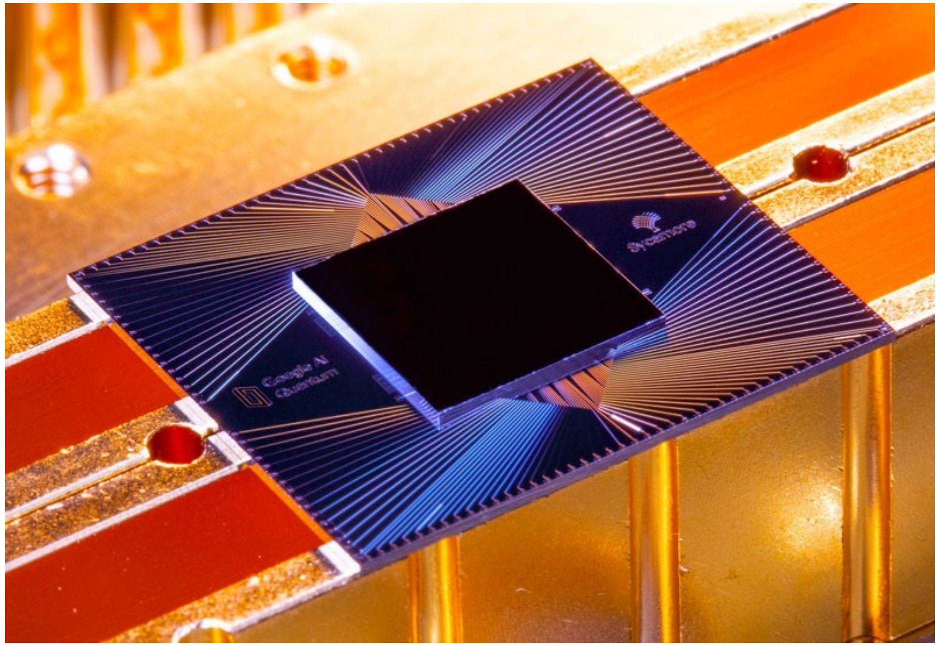
arxiv: 2102.01738

(see related talk by Kondo, Mori and Movassagh)

Quantum supremacy experiments

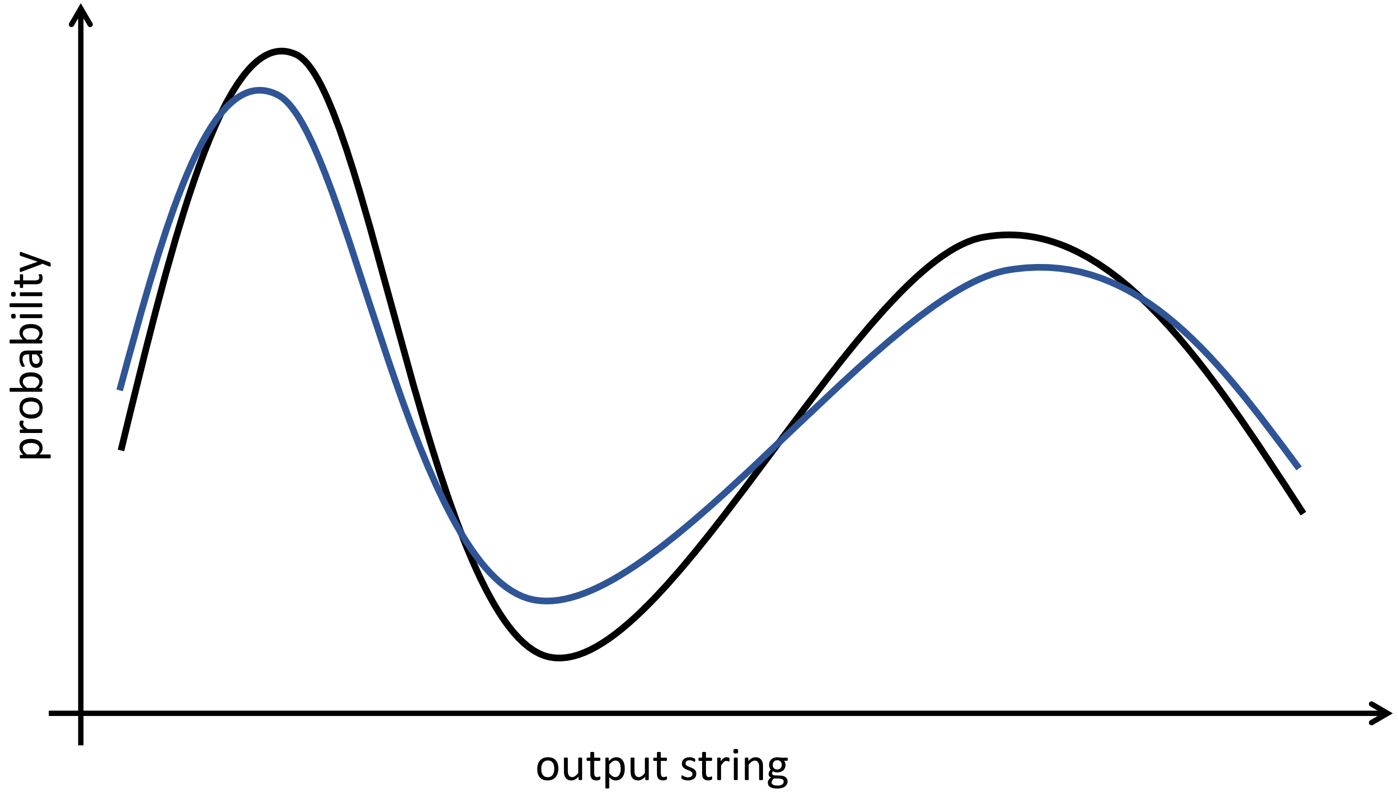


Random Circuit Sampling (Google Sycamore)

BosonSampling (USTC Jiuzhang)

This talk: improved complexity-theoretic evidence that these tasks are hard for classical computers

Evidence of hardness for quantum supremacy experiments

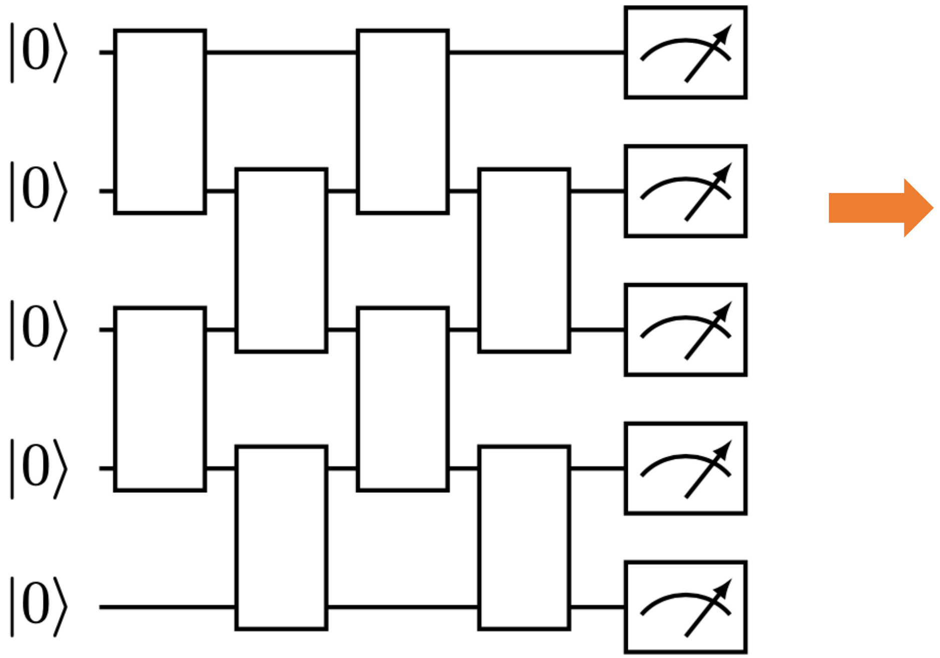


- Goal: prove hardness of sampling with small total variation distance from the ideal distribution
- *ideal regime*

- Limitations: cannot prove hardness even in this idealized setting due to insufficient robustness

First result: significantly improve the robustness of prior hardness results

Computational model: ideal regime



11011000001011111000100110110111100111101001011110101,
11110000101010101110111011100000000010001111110111010001,
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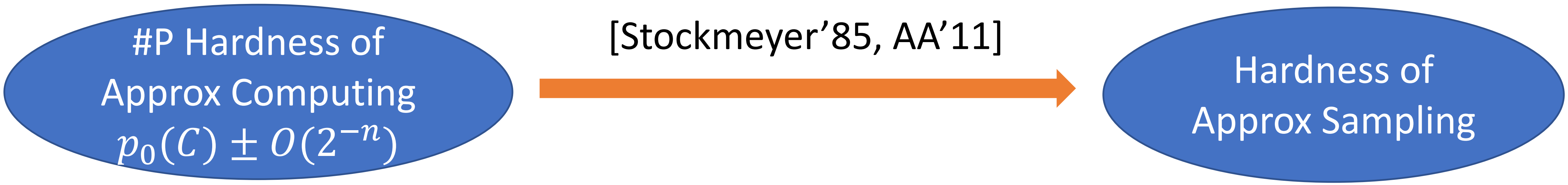
ideal regime:

Goal: Prove it is hard to sample from a distribution that is close to the ideal distribution in total variation distance

Evidence for hardness of sampling



Evidence for hardness of sampling

Goal: #P hardness of computing $|\langle 0|C|0\rangle|^2 \pm O(2^{-n})$

Robustness to additive imprecision

$\exp(-O(\text{poly}(n)))$ $O(2^{-n})$

0

Previous work [AA'11,BFNV'19,Mov'20]

First result

Goal

First result: improved robustness in the ideal regime

Task	Previous result	Our result	Goal	Remark
Random circuit sampling (n qubits, m gates)	$2^{-\mathrm{poly}(m)}$ [BFNV'19] $\exp(-O(m^3))$ [Mov'20]	$\exp\left(-O(m\log m)\right)$ See also related work of [KMM'21]	$O(2^{-n})$	

Robustness to additive imprecision (random circuit sampling)

First result: improved robustness in the ideal regime

Task	Previous result	Our result	Goal	Remark
Random circuit sampling (n qubits, constant depth)	$2^{-\mathrm{poly}(n)}$ [BFNV'19] $\exp(-O(n^3))$ [Mov'20]	$\exp\big(-O(n\log n)\big)$ See also related work of [KMM'21]	$O(2^{-n})$	For constant depth circuits, tight up to $O(\log n)$ factor in the exponent

Robustness to additive imprecision (random circuit sampling)

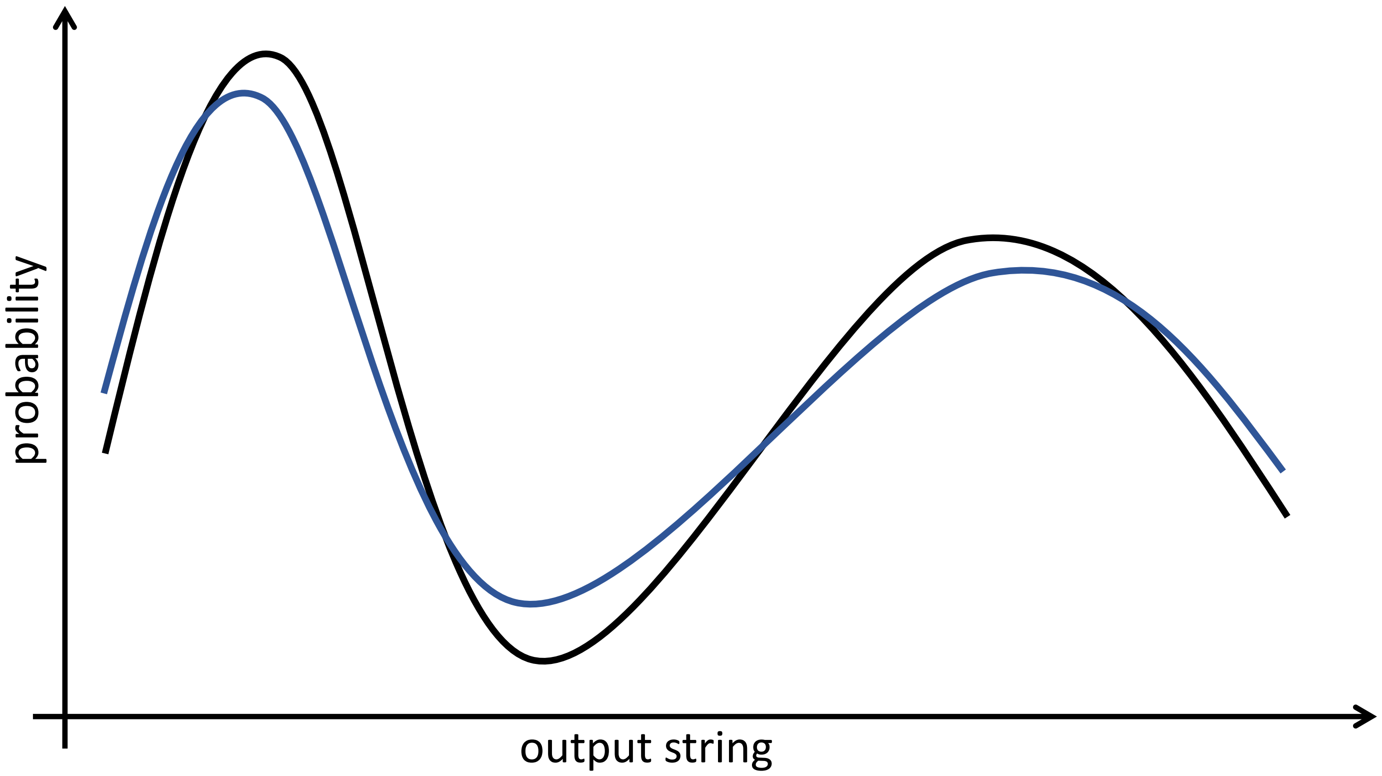
0	$2^{-\mathrm{poly}(n)}$ [BFNV'19]	$\exp(-O(m^3))$ [Mov'20]	**First result** (Also see [KMM'21])	Goal

First result: improved robustness in the ideal regime

Task	Previous result	Our result	Goal	Remark
Random circuit sampling (n qubits, constant depth)	$2^{-\mathrm{poly}(n)}$ [BFNV'19] $\exp(-O(n^3))$ [Mov'20]	$\exp\left(-O(n\log n)\right)$ See also related work of [KMM'21]	$O(2^{-n})$	For constant depth circuits, tight up to $O(\log n)$ factor in the exponent
BosonSampling (n photons, n^2 detectors)	$\exp(-O(n^4))$ [AA'11]	$\exp(-6n\log n)$	$\exp(-n\log n)$	Tight up to constant factor in the exponent

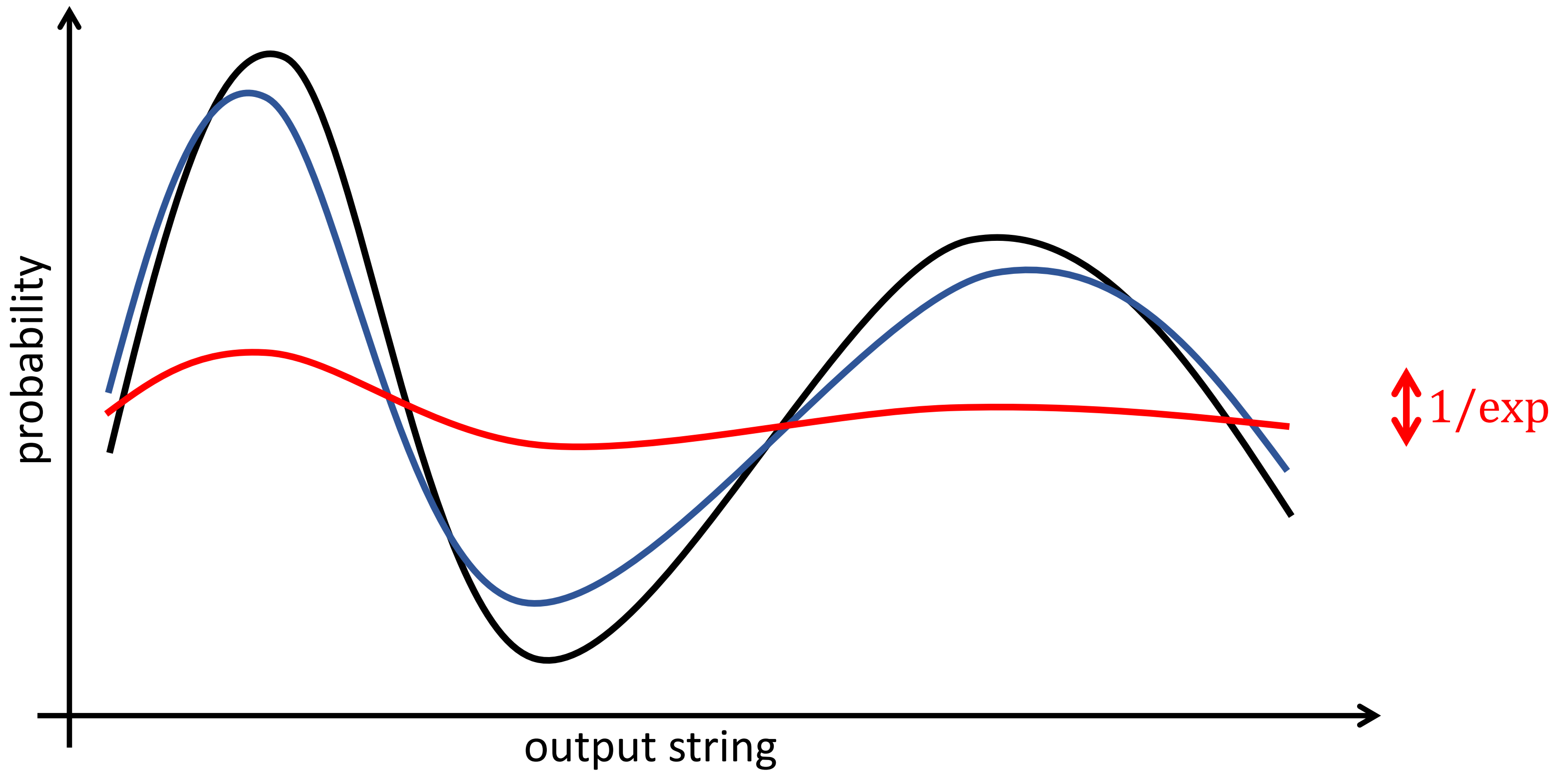
Robustness to additive imprecision (BosonSampling)

Theory vs. Experiment



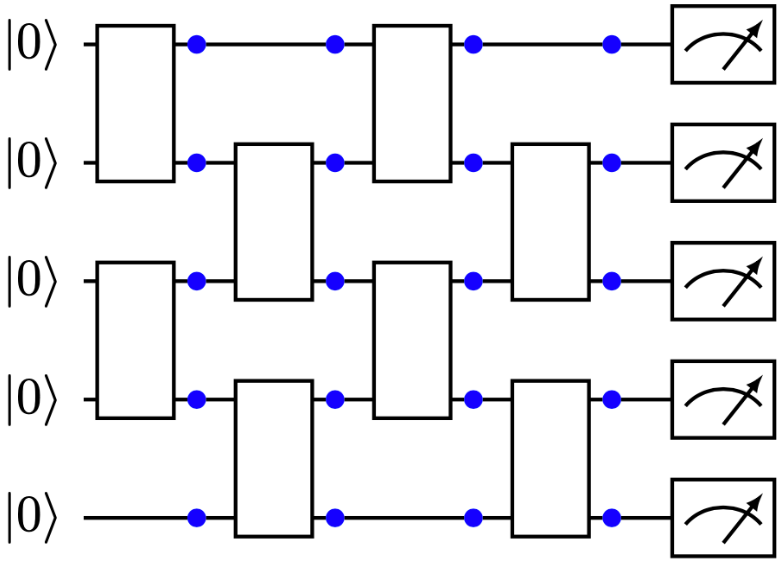
ideal regime: Goal is to prove hardness of sampling from a distribution that is very close to the ideal distribution

Theory vs. Experiment



High noise regime: in experiments we only observe a tiny deviation from uniform along the correct direction

New model: high-noise regime



Blue: noise channels

1101100000010111111000100110110111100111101001011110101,
111100001010101011101110111000000000100011111011101001,
00010001011010100010110010000101000000110100001010010…

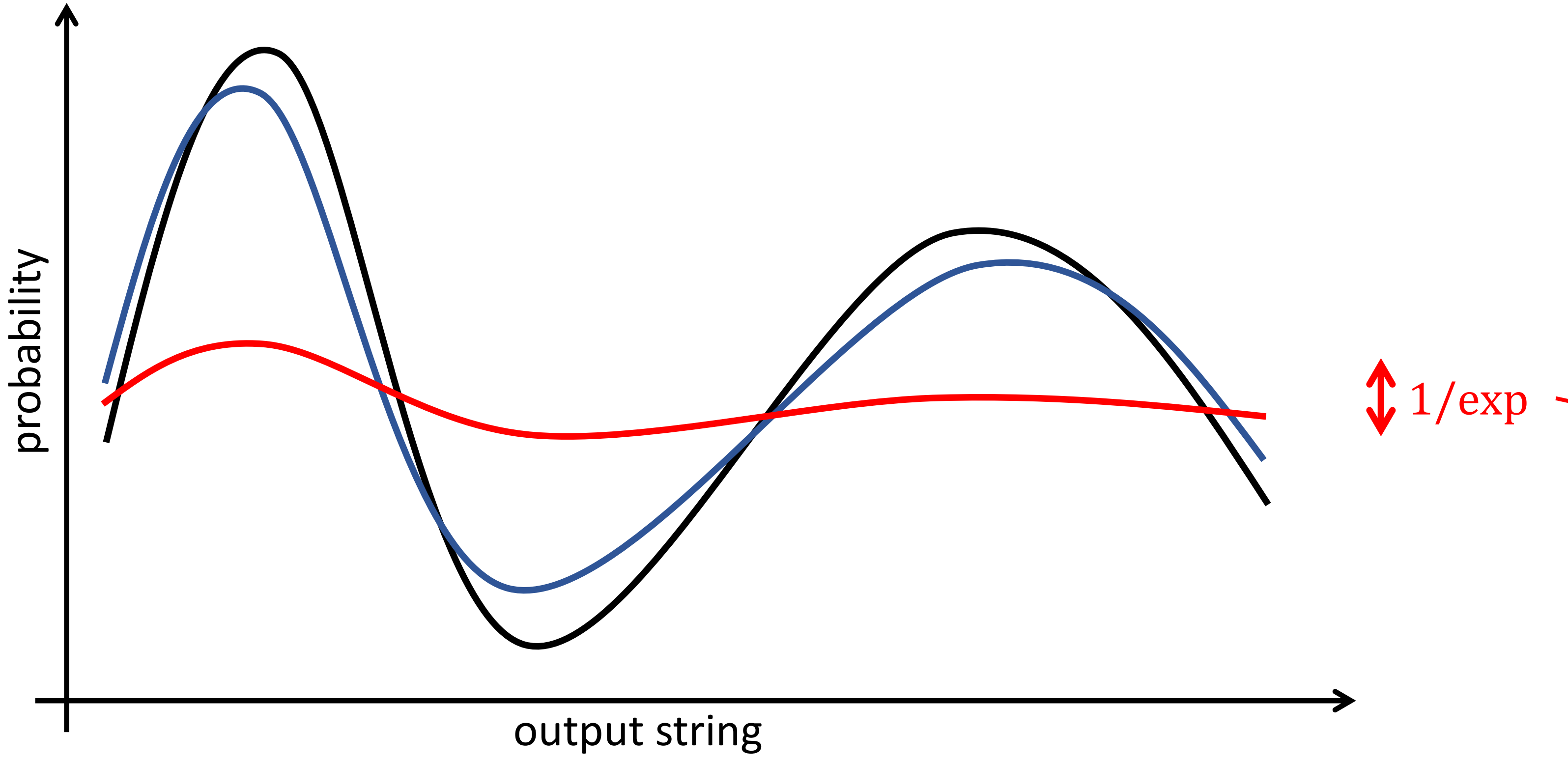
High-noise regime:

Is it hard to sample from the noisy distribution?

Is it hard to compute the output probability of noisy circuits?

Is there any signal of hardness in these highly noisy distributions?

New model: high-noise regime

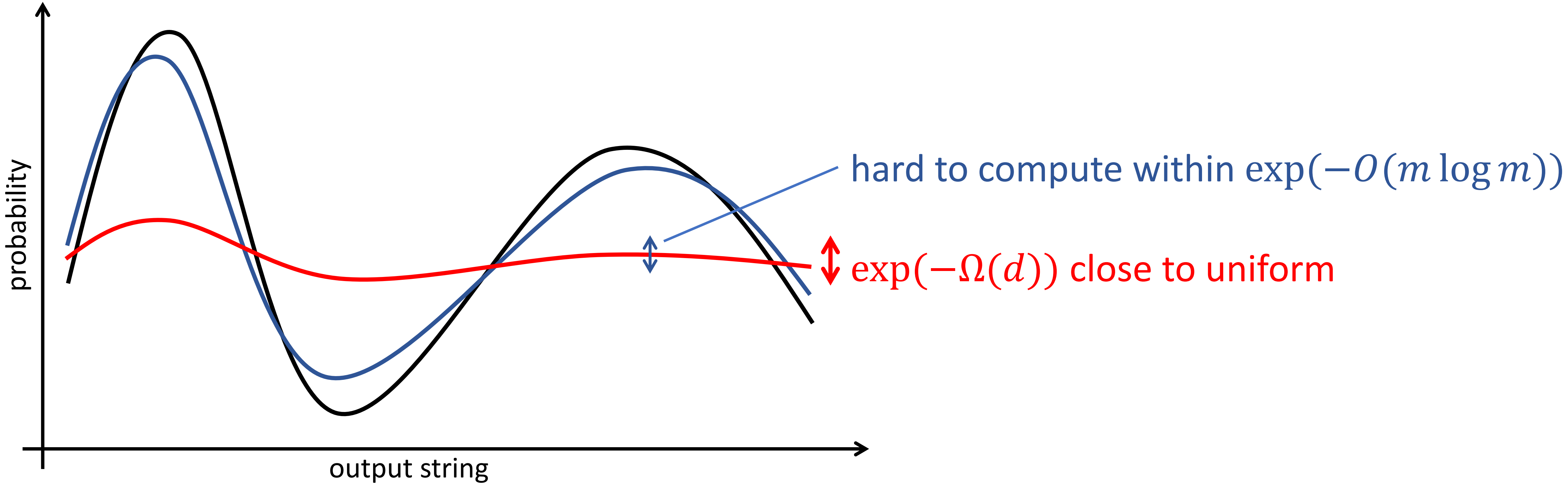


Noisy random circuits converge to uniform exponentially quickly

[ABOIN'96, GD'18, Deshpande et al'21]

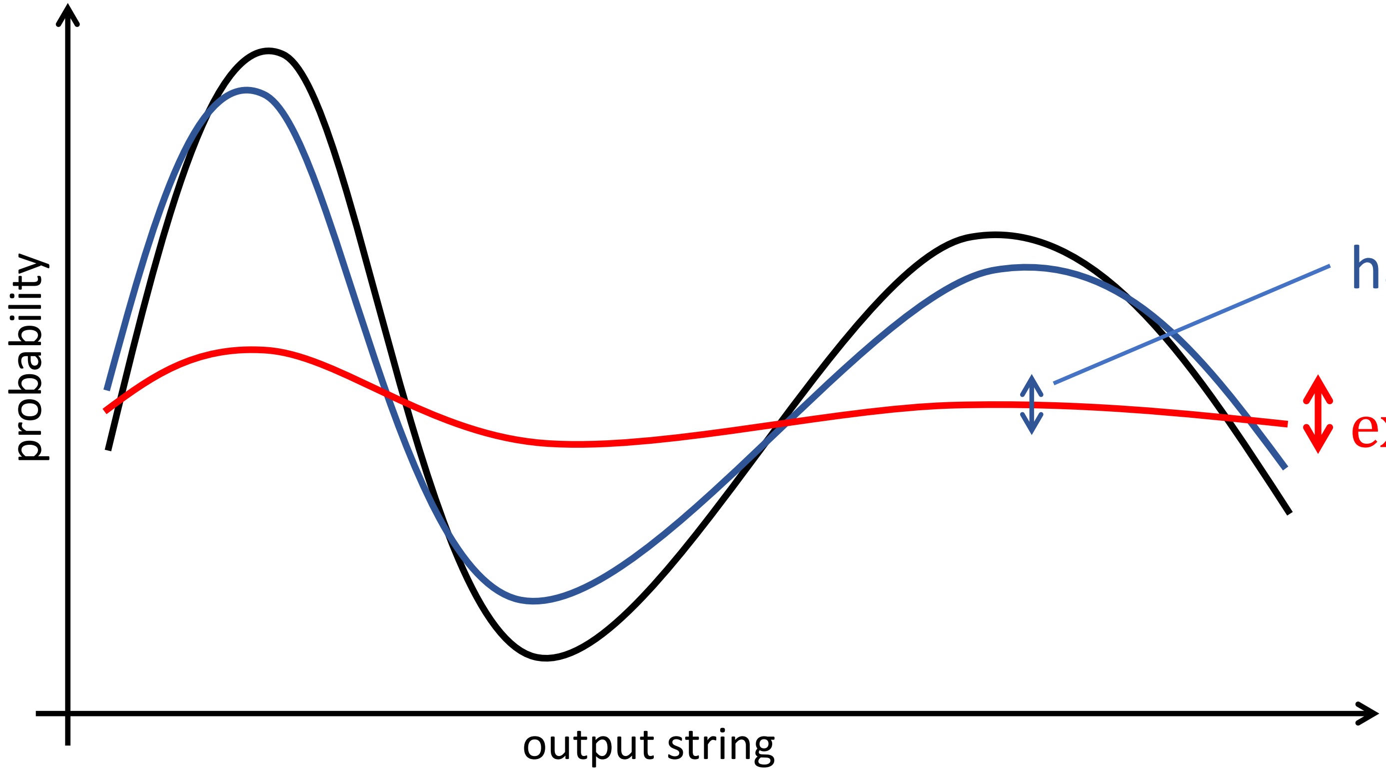
Second result: these tiny signals remain hard to *compute*

Second result: evidence of hardness in the high-noise regime



Second result: these tiny signals remain hard to *compute*

Second result: evidence of hardness in the high-noise regime



hard to compute within $\exp(-O(m \log m))$

$\exp(-\Omega(d))$ close to uniform

Cannot improve our high-noise result much further for deep circuits, due to the exponential convergence to uniform

Second result: these tiny signals remain hard to *compute*

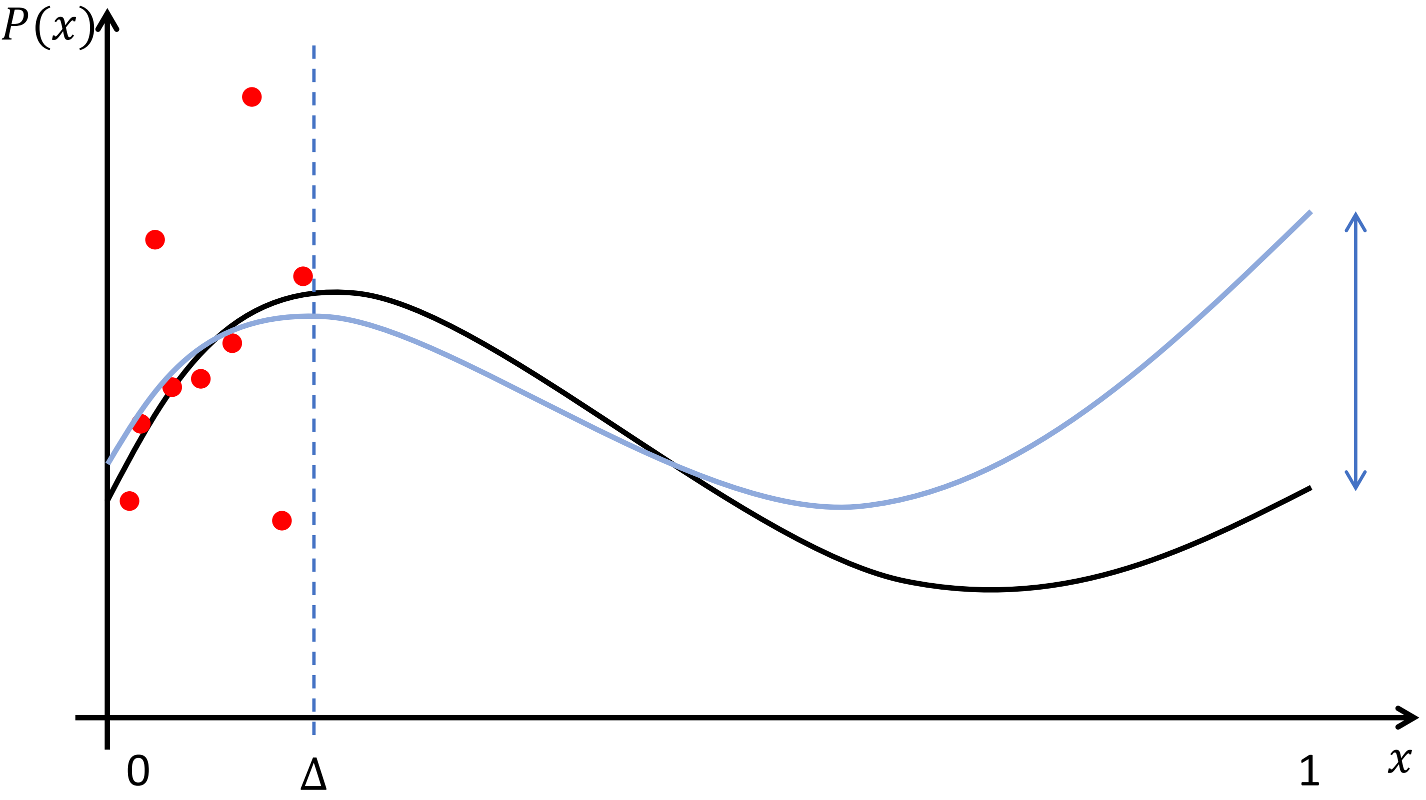
Proof sketch

An algorithm for computing the output probability of random circuits

↓ Polynomial structure [AA'11, BFNV'19, Mov'20]

An algorithm for computing the output probability of any circuit

Proof techniques: first result



- The problem reduces to polynomial interpolation on noisy data points [AA’11, BFNV’19, Mov’20]
- We develop a robust Berlekamp-Welch argument that
  - simplifies the proof
  - tolerates more errors
  - reduces the extrapolation error

Proof techniques: second result

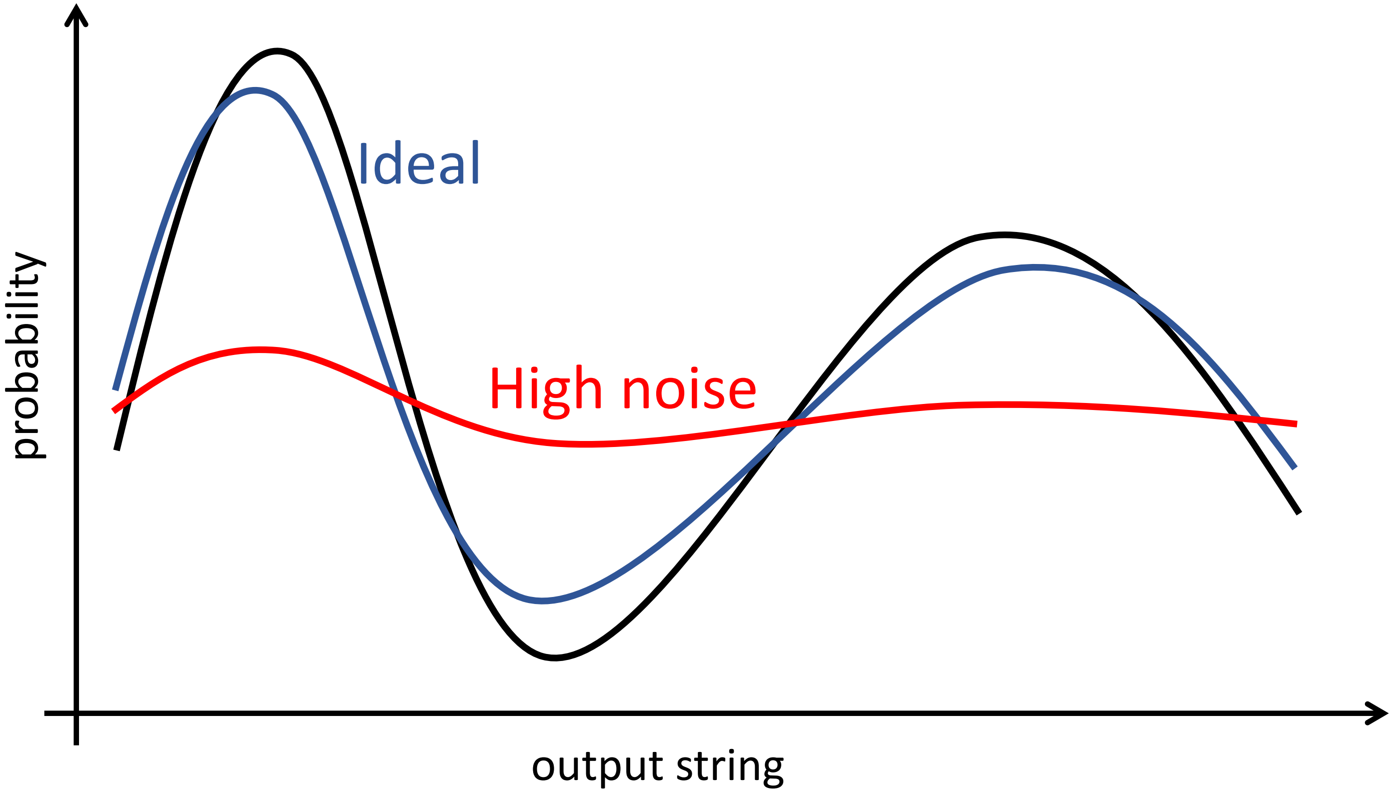
Hardness of ideal circuits → Hardness of noisy circuits

Linearity of noise channels

Preserves the polynomial structure

- The same worst to average case reduction techniques also apply to the high noise setting
- *Q: what about worst case hardness?*
- A: error detection [Fujii'16]

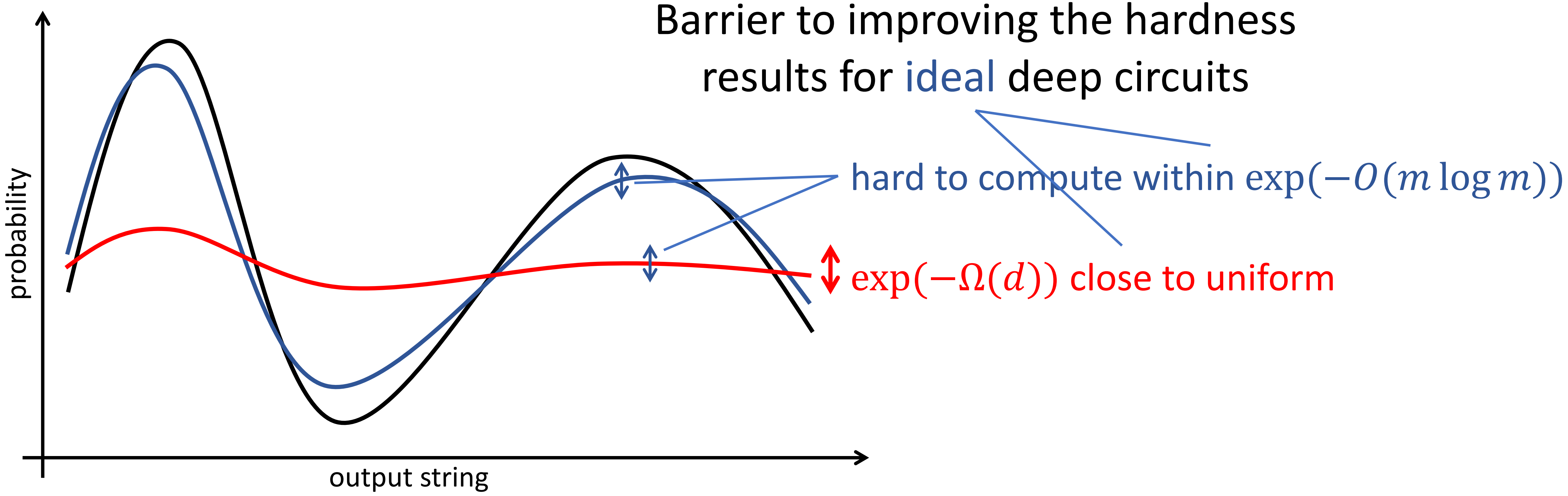
Summary of our results



- Our result: we substantially improve the robustness of prior hardness results in the ideal setting
- Our result: we give initial evidence of hardness with exponentially decreasing fidelity

High-noise result implies barrier to improving ideal result

Barrier to improving the hardness results for ideal deep circuits



Barriers to proving hardness of sampling (in the ideal regime)

Random circuit sampling
- Noise barrier [This work]
- Depth barrier [Napp et al'20]
- Polynomial interpolation barrier [AA'11]

BosonSampling
- Polynomial interpolation barrier [AA'11]
- *Q: do noise and depth barriers apply?*

- Our result: $\exp\left(-O(n \log n)\right)$
- Goal: $O(2^{-n})$

- Our result: $\exp(-6n \log n)$
- Goal: $\exp(-n \log n)$